THE $\pi\gamma$ TRANSITION FORM FACTOR AND THE PION WAVE FUNCTION

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arXiv:hep-ph/9605264v1 9 May 1996

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Abstract

The pion wave function is discussed in the light of the recent CLEO data on the $\pi\gamma$ transition form factor. It turns out that the wave function is close to the asymptotic form whereas wave functions strongly concentrated in the end-point regions are disfavoured. Consequences for other exclusive quantities, as for instance the pion's electromagnetic form factor, are also discussed.

The theoretical description of large momentum transfer exclusive reactions is based on a factorization of long- and short-distance physics. The latter physics is contained in the so-called hard scattering amplitude to be calculated within perturbation theory. Universal, process-independent (light cone) wave functions interpolating between hadronic and partonic degrees of freedom, comprise the long-distance physics. The wave functions are not calculable with sufficient degree of accuracy at present. However, for the pion which is the hadron of interest in this letter, the valence Fock state wave function is theoretically rather well constrained. The main uncertainty of the wave function lies in the x-dependent part of it, the so-called distribution amplitude $\phi(x)$. x denotes the usual momentum fraction the valence quark carries.

Previous studies of the pion's electromagnetic form factor as well as other large momentum transfer exclusive reactions, as for instance $\gamma\gamma \to \pi\pi$, seemed to indicate that the pion distribution amplitude is much broader than the so-called asymptotic one, $\phi_{AS}(x) = 6x(1 - x)$. Chernyak and Zhitnitsky [1] proposed such a broad distribution amplitude which is strongly end-point concentrated and leads to a leading twist contribution to the pion's electromagnetic form factor in apparently fair agreement with the admittedly poor data [2]. This result is, however, obtained at the expense of the dominance of contributions from the end-point regions, $x \to 0$ or 1, where the use of perturbative QCD is unjustified as has been pointed out by several authors [3, 4]. Now the prevailing opinion is that the pion's electromagnetic form factor is controlled by soft physics (e.g. the overlap of the initial and final state wave functions, occasionally termed the Feynman contribution) for momentum transfer less than about 10 GeV^2 [3, 5, 6, 7].

There is another exclusive quantity namely the $\pi\gamma$ transition form factor which, for experimental and theoretical reasons, allows a more severe test of our knowledge of the pion wave function than the pion's electromagnetic form factor. Recently the $\pi\gamma$ form factor has been measured in the momentum transfer region from 2 to 8 GeV^2 [8] with rather high precision. Together with previous CELLO measurement [9] we now have at our disposal much better data above 1 GeV^2 for the $\pi\gamma$ transition form factor than for the pion form factor. From the theoretical point of view the analysis of the $\pi\gamma$ transition form factor is much simpler than that of the pion form factor: It is, to lowest order, a QED process, QCD only provides corrections of the order of 10% in the momentum transfer region of interest [10]. The difficulties with the end-point regions where the gluon virtuality becomes small, do not occur. Higher Fock state contributions, suppressed by powers of α_s/Q^2, are expected to be small (see the discussion in [11]). Moreover, and in contrast to the pion form factor, the Feynman contribution, which may arise through vector meson dominance, is presumably very small due to a helicity mismatch.

The purpose of this letter is to extract information on the pion wave function from a perturbative analysis of the $\pi\gamma$ form factor. It will turn out that the new CLEO data [8] allow a fairly precise determination of that wave function. It will

also be argued that this wave function leads to a consistent description of the pion's electromagnetic form factor and its structure function. It should be noted that this letter is an update of previous work [12]. The very important CLEO data [8] were not yet available in [12]. We also point out that the $\pi\gamma^*$ transition form factor is investigated in [13].

Let us begin with the parameterization of the soft valence Fock state wave function, i.e. the full wave function with the perturbative tail removed from it. This is the object required in a perturbative calculation [14]. Following [5] the soft wave function is written as

$$\hat{\Psi}_0\left(x, \mathbf{b}, \mu_F\right) = \frac{f_\pi}{2\sqrt{6}}\,\phi(x, \mu_F)\,\hat{\Sigma}\left(\sqrt{x(1-x)}\mathbf{b}\right). \tag{1}$$

μ_F is the scale at which soft and hard physics factorize [14, 15, 16]. The wave function is subject to the auxiliary conditions

$$\hat{\Sigma}(0) = 4\pi, \qquad\qquad \int_0^1 dx\,\phi(x, \mu_F) = 1. \tag{2}$$

$\mathbf{b}$ is the quark-antiquark separation in the transverse configuration space and is canonically conjugated to the usual transverse momentum $\mathbf{k}_\perp$. It is advantageous to work in the transverse configuration space because the Sudakov factor, to be discussed later, is only derived in that space (see [16]). The parameterization (1) automatically satisfies the constraint from the process $\pi^+ \to \mu^+\nu_\mu$ [14] which relates the wave function at the 'origin of the configuration space' to the pion decay constant $f_\pi(= 130.7\text{ MeV})$. On the assumption of duality properties, one can derive constraints on x and $k_\perp$ moments[3] of the wave function (e.g. $\langle x^n \rangle = \int dx\, x^n\, \phi(x)$) within the operator product expansion framework [17]. These constraints can be combined into the following conditions on the momentum space wave function:

- i)   The distribution amplitude has simple zeroes at $x \to 0, 1$.
- ii) The $\mathbf{k}_\perp$-dependence of the wave function $\Psi_0\left(x, \mathbf{k}_\perp\right)$ comes exclusively in the combination $k_\perp^2/x(1-x)$ at $x \to 0, 1$.
- iii) At large $k_\perp$ the wave function falls off faster than any power of $k_\perp$.

The simplest function matching the conditions ii) and iii) is the Gaussian

$$\hat{\Sigma}\left(\sqrt{x(1-x)}\,\mathbf{b}\right) = 4\pi \exp\left[-\frac{x(1-x)\,b^2}{4a^2}\right] \tag{3}$$

where a is the transverse size parameter. This Gaussian will be used subsequently.

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[3] There is a subtlety: The kinematical transverse momentum of the parton is not the same object as $k_\perp$ defined through the moments. In this letter we will assume that both are one and the same variable. This assumption corresponds to summing up soft gluon corrections, i.e. to higher twist contributions.

The distribution amplitude is subject to evolution and can be expanded over Gegenbauer polynomials $C_n^{3/2}$, the eigenfunctions of the (leading order) evolution equation for mesons

$$\phi(x, \mu_F) = \phi_{AS}(x) \left[1 + \sum_{n=2,4,\ldots}^{\infty} B_n \left(\frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)} \right)^{\gamma_n} C_n^{3/2}(2x-1) \right]. \qquad (4)$$

α_s is the strong coupling constant and μ_0 is a typical hadronic scale for which we choose 0.5 GeV throughout. Charge conjugation invariance requires the odd n expansion coefficients B_n to vanish. Since the γ_n are positive fractional numbers increasing with n (e.g. $\gamma_2 = 50/81$) any distribution amplitude evolves into ϕ_{AS} asymptotically, i.e. for $\ln(Q/\mu_0) \to \infty$; higher order terms in (4) are gradually suppressed. The asymptotic distribution amplitude itself shows no evolution. This property of ϕ_{AS} no more holds if evolution is treated in next-to-leading order [18]. As we are going to show below the asymptotic distribution amplitude in combination with the Gaussian (3) provides very good results for the $\pi\gamma$ form factor and also leads to a consistent description of other exclusive reactions involving pions. In order to quantify the amount of deviations from the asymptotic distribution amplitude still allowed by the CLEO data, contributions from the second Gegenbauer polynomial will be permitted in the analysis and limits for the strength of the coefficient B_2 will be extracted. For the purpose of comparison the $\pi\gamma$ form factor will be evaluated with the Chernyak-Zhitnitsky (CZ) distribution amplitude [1] which is defined by $B_2 = 2/3$ and $B_n = 0$ for $n > 2$.

For a given distribution amplitude there is only one free parameter in the wave function, namely the transverse size parameter a. It can be fixed by using a constraint derived from the process $\pi^0 \to \gamma\gamma$ [14]:

$$\int dx\, d^2\mathbf{b}\, \hat{\Psi}_0(x, \mathbf{b}) = \frac{\sqrt{6}}{f_\pi}. \qquad (5)$$

Although not at the same level of rigour as the other constraints (because of approximations made in the case when one photon couples 'inside' the pion wave function) (5) still provides a value for the parameter a which comes up to our expectations for the transverse size of the pion. In particular for the asymptotic wave function one finds a value of 861 MeV for a, which corresponds to a value of 367 MeV for the root mean square transverse momentum. The probability of the valence Fock state amounts to 0.25 in this case.

Now the wave function is fully specified and we can turn to the calculation of the $\pi\gamma$ form factor. Inspection of the $\pi\gamma$ form factor data [8, 9] (see Fig. 1) reveals a Q^2 dependence somewhat stronger than predicted by dimensional counting. Hence, higher twist contributions do not seem to be negligible in the momentum transfer region from 1 to 8 GeV^2. The modified perturbative approach proposed by Sterman and collaborators [16, 19] allows to calculate some power corrections to the leading twist term. In that approach the transverse momentum dependence of the hard scattering amplitude is retained and Sudakov suppressions are

taken into account in contrast to the standard approach [15]. Applications of the modified perturbative approach to the pion's and nucleon's electromagnetic form factors [5, 19, 20] revealed that the perturbative contributions to these form factors can self-consistently (in the sense that the bulk of the contributions is accumulated in regions where the strong coupling α_s is sufficiently small) be calculated. It turned out, however, that the perturbative contributions are too small as compared with the data. Therefore, in the experimentally accessible range of momentum transfer, these form factors are controlled by soft physics. Higher order perturbative corrections and/or higher Fock state contributions seem too small in order to account for the large discrepancies between the lowest order perturbative contributions and the data for the elastic form factors (see [21] for the pion case).

For the reasons discussed in the introduction the $\pi\gamma$ transition form factor represents an exceptional case for which we can expect perturbation theory to work for Q^2 larger than about 1 or 2 GeV^2. Adapting the modified perturbative approach [16, 19] to the case of $\pi\gamma$ transitions, we write the corresponding form factor as

$$F_{\pi\gamma}\left(Q^2\right) = \int dx \, \frac{d^2\mathbf{b}}{4\pi} \hat{\Psi}_0\left(x, -\mathbf{b}, \mu_F\right) \, \hat{T}_H\left(x, \mathbf{b}, Q\right) \exp\left[-S\left(x, b, Q\right)\right]. \quad (6)$$

This convolution formula can formally be derived using the methods described in detail by Botts and Sterman [16]. $\hat{T}_H$ is the Fourier transform of the momentum space hard scattering amplitude to be calculated, to lowest order, from the Feynman graphs shown in Fig. 2. It reads

$$\hat{T}_H\left(x, \mathbf{b}, Q\right) = \frac{2}{\sqrt{3}\pi} K_0\left(\sqrt{1-x}\, Q\, b\right) \quad (7)$$

where K_0 is the modified Bessel function of order zero. The Sudakov exponent S in (6), comprising those gluonic radiative corrections (in next-to-leading-log approximation) not taken into account in the evolution of the wave function, is given by

$$S(x, b, Q) = s(x, b, Q) + s(1-x, b, Q) - \frac{4}{\beta_0} \ln \frac{\ln\left(\mu/\Lambda_{QCD}\right)}{\ln\left(1/b\,\Lambda_{QCD}\right)} \quad (8)$$

where a Sudakov function s appears for each quark line entering the hard scattering amplitude. The last term in (8) arises from the application of the renormalization group equation ($\beta_0 = 11 - \frac{2}{3}n_f$). A value of 200 MeV for Λ_{QCD} is used and μ is taken to be the largest mass scale appearing in the hard scattering amplitude, i.e. $\mu = \max\left(\sqrt{1-x}\,Q, 1/b\right)$. For small b there is no suppression from the Sudakov factor; as b increases the Sudakov factor decreases, reaching zero at $b = 1/\Lambda_{QCD}$. For even larger b the Sudakov is set to zero. The Sudakov function s has been calculated by Botts and Sterman [16] using resummation techniques;

its explicit form can be found in [22]. Due to the properties of the Sudakov factor any contribution is damped asymptotically, i.e. for $\ln(Q^2/\mu_0^2) \to \infty$, except those from configurations with small quark-antiquark separations and, as can be shown, the limiting behaviour $F_{\pi\gamma} \to \sqrt{2}f_\pi/Q^2$ emerges, a result which as been derived previously [23, 24]. b plays the role of an infrared cut-off; it sets up the interface between non-perturbative soft gluon contributions - still contained in the hadronic wave function - and perturbative soft gluon contributions accounted for by the Sudakov factor. Hence, the factorization scale μ_F is to be taken as $1/b$.

Employing the asymptotic wave function, (1), (3) and ϕ_{AS}, we find, from (6), the numerical results for the $\pi\gamma$ transition form factor displayed in Fig. 1. Obviously there is very good agreement with the data [8, 9] above $Q^2 \simeq 1 \text{ GeV}^2$. At 8 GeV^2 about 85% of the asymptotic value has been reached. We emphasize that there is no free parameter in our approach to be fitted to the data once the wave function is chosen and the transverse size parameter is fixed through (5). For comparison we also show in Fig. 1 results obtained with the CZ wave function, (1), (3) and ϕ_{CZ} ($B_2 = 2/3$, $B_n = 0$ for $n > 2$ in (4)). That prediction overshoots the data markedly. Of course, the experimental errors allow slight modifications of the asymptotic wave function. In order to give a quantitative estimate of the allowed modifications we fit the expansion coefficient B_2 to the data assuming $B_n = 0$ for $n > 2$ and choosing again $\mu_0 = 0.5$ GeV. For each value of B_2 the transverse size parameter a is fixed through (5). A best fit to the data above 1 GeV^2 provides $B_2^{mpa} = -0.006 \pm 0.014$ (with $a = 864$ MeV), i.e. a value compatible with zero. In [14] a modification of the asymptotic wave function is proposed where ϕ_{AS} is multiplied by the exponential $\exp\left[-m_q^2 a^2/x(1-x)\right]$. The parameter m_q represents a constituent quark mass of, say, 330 MeV. A similar wave function is constructed by Dorokhov [7] from the helicity and flavour changing instanton force. Although wave functions of this type contradict the constraint i) derived by Chibisov and Zhitnitsky [17], they cannot be excluded absolutely since the constraint i) is obtained under a duality assumption the validity of which is not guaranteed. In any case we have convinced ourselves that the wave function given in [14] provides similarly good results for the $\pi\gamma$ form factor as the asymptotic wave function itself. This is not a surprise since both the wave functions differ from each other only in the end-point regions. Contributions from these regions are strongly suppressed by the Sudakov factor.

It is instructive to compare the leading twist result for the $\pi\gamma$ form factor [15, 23]

$$F_{\pi\gamma}\left(Q^2\right) = \frac{\sqrt{2}}{3}\langle x^{-1}\rangle\frac{f_\pi}{Q^2}\left[1 + \frac{\alpha_s(\mu_R)}{2\pi}K(Q,\mu_R) + \mathcal{O}(\alpha_s^2)\right] \qquad (9)$$

with the data above $Q^2 \simeq 3 \text{ GeV}^2$ which, within errors, are just compatible with a Q^2-dependence according to dimensional counting. μ_R represents the renormalization scale. The factor K depends on the distribution amplitude. Using the

expansion (4), one finds for the x^{-1} moment of the distribution amplitude

$$\langle x^{-1} \rangle = 3 \left[1 + B_2 \left(\frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)} \right)^{50/81} + ... \right]. \tag{10}$$

Neglecting again terms with $n > 2$ and also the α_s corrections in (9), we obtain a value of -0.39 ± 0.05 for B_2^{LO} from a fit to the data ($\mu_F = Q$ in this case). The face value of B_2^{LO} corresponds to $\langle x^{-1} \rangle^{LO} = 2.39$ (at $Q^2 = 8$ GeV^2) which is to be contrasted with the values of 3 and 4.01 (at 8 GeV^2) for the asymptotic and the CZ distribution amplitude respectively.

Braaten [10] has calculated the α_s corrections (in the $\overline{MS}$ scheme) in (9). His analysis is however incomplete in so far as only the α_s corrections to the hard scattering amplitude have been considered but the corresponding corrections to the kernel of the evolution equation for the pion's distribution amplitude were ignored. As has been shown by Müller [18] recently in next-to-leading order the evolution provides logarithmic modifications in the end-point regions for any distribution amplitude, i. e. for the asymptotic one too. An estimate however reveals that the modifications of the evolution behaviour in next-lo-leading order are very small for the asymptotic distribution amplitude (α_s evaluated in two-loop with $\Lambda_{n_F=3}^{\overline{MS}} = 200$ MeV), and can safely be neglected here. For the CZ distribution amplitude these effects seem to be somewhat larger than for the asymptotic one but still the total α_s corrections are dominated by those to the hard scattering amplitude. The α_s corrections amount to -10% in the case of the CZ distribution amplitude. Hence, also in the leading twist analysis in next-to-leading order the CZ distribution amplitude is clearly at variance with the data. Next we want to determine the expansion coefficient B_2 in the next-to-leading order leading twist analysis in order to quantify the deviations from the asymptotic distribution amplitude required by the $F_{\pi\gamma}$ data. For this purpose we evaluate the K factor in (9) from the expressions for the α_s corrections given in [10] ($\mu_R = \mu_F = Q$) and neglect the modifications of the evolutions in next-to-leading order. From a fit of (9) to the form factor data we find $B_2^{NLO} = -0.17 \pm 0.05$ corresponding to $\langle x^{-1} \rangle^{NLO} = 2.74$ at $Q^2 = 8$ GeV^2. According to what we said above such small a value of B_2 will not be altered substantially by the modifications of the evolution behaviour to that order. Thus, the leading twist analysis requires a distribution amplitude which is a little narrower than the asymptotic one. In the modified perturbative approach, on the other hand, the asymptotic wave function works well since the QCD corrections condensed in the Sudakov factor, and the transverse degrees of freedom provide the required Q^2-dependent suppressions. It is to be stressed that the Sudakov factor already takes into account the leading and next-to-leading logs of the α_s corrections.

Other models, applicable at large as well as at low Q^2, provide a parameterization of the form factor as

$$F_{\pi\gamma}\left(Q^2\right) = A/(1 + Q^2/s_0). \tag{11}$$

Thus, Brodsky and Lepage [23] propose that parameterization as an interpolation between the two limits, $F_{\pi\gamma}(Q^2 = 0) = A = (2\sqrt{2}\pi^2 f_\pi)^{-1}$ known from current algebra and the limiting behaviour $\sqrt{2} f_\pi Q^{-2}$. Hence, $s_0 = 4\pi^2 f_\pi^2 = 0.67$ GeV^2 in that model. The interpolation formula works rather nicely, its Q^2-dependence is similar to that one predicted by the modified perturbative approach. The vector meson dominance model leads to (11) under the neglect of contributions from the ϕ meson and by ignoring the small mass difference between the ρ and ω meson. The constant A is related to

$$A = \frac{g_{\pi\rho\gamma}}{f_\rho} + \frac{g_{\pi\omega\gamma}}{f_\omega} \tag{12}$$

in the vector meson dominance model. s_0 equals m_ρ^2 where m_ρ is the ρ meson mass. Inserting the known values of coupling constants [25], one finds for A a value of 0.269 ± 0.019 in agreement with the current algebra value. The vector meson dominance model is in accord with the present data although its asymptotic value ($A\, m_\rho^2 \simeq 0.16$) differs from our one. A QCD sum rule analysis [26] also provides results similar to (11). In order to discriminate among the various models data extending to larger values of momentum transfer and/or with smaller errors as the present ones are needed.

Let us now turn to the discussion of the implications of our findings, namely that the perturbative analysis of the $\pi\gamma$ transition form factor requires the asymptotic pion wave function and apparently excludes strongly end-point concentrated wave functions like the one proposed by Chernyak and Zhitnitsky. Since the wave functions are universal, process-independent objects they should also be used in other large momentum transfer exclusive reactions involving pions, as for example the electromagnetic form factor of the pion or $\gamma\gamma \to \pi\pi$. As is well-known the leading twist results are only in agreement with experiment provided an end-point concentrated wave function respectively distribution amplitude is utilized[4]. This apparent agreement with experiment is, as we already mentioned, only obtained at the expense of strong contributions from the soft end-point regions where the use of perturbation theory is unjustified. This is to be contrasted with the modified perturbative approach where the end-point regions are strongly suppressed and a theoretically self-consistent perturbative contribution is obtained. However, as shown in [5] for the case of the pion form factor, the perturbative contributions evaluated with both the wave functions, the asymptotic one and the CZ one, are too small as compared with the data. At this point we remind the reader of the fact that the pion form factor also gets contributions from the overlap of the initial and final state soft wave functions $\hat{\Psi}_0$ (1). Formally the perturbative contribution to the pion form factor represents the overlap of the large momentum tails of the wave functions while the overlap of the soft parts of

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[4] Taking the moment $\langle x^{-1} \rangle$ which represents a soft, process-independent parameter, from our leading twist analysis of $F_{\pi\gamma}$, we find a value for the pion form factor too small as compared to the admittedly poor data (note: $F_\pi \sim \langle x^{-1} \rangle^2$) even when α_s corrections are considered [21].

the wave functions is customarily assumed to be negligible at large Q. Examining the validity of that presumption by estimating the Feynman contribution from the asymptotic wave function, one finds results of appropriate magnitude to fill in the gap between the perturbative contribution and the data of Ref. [2]. The results exhibit a broad flat maximum which, for momentum transfers between 3 and about 15 GeV^2, simulates the dimensional counting behaviour[5]. For the CZ wave function, on the other hand, the Feynman contribution exceeds the data significantly. Similar large Feynman contributions have also been obtained by other authors [3, 6, 7]. Thus, the small size of the perturbative contribution to the elastic form factor finds a comforting although model-dependent explanation, a fact which has been pointed out by Isgur and Llewellyn Smith [3] long time ago.

The structure function of the pion offers another possibility to test the wave function against data. As has been shown in [14] the parton distribution functions are determined by the Fock state wave functions. Since each Fock state contributes through the modulus squared of its wave function integrated over transverse momenta up to Q and over all fractions x except those pertaining to the type of parton considered, the contribution from the valence Fock state should not exceed the data of the valence quark structure function. As discussed in [12, 27] the asymptotic wave function respects this inequality while the CZ one again fails dramatically.

To conclude the asymptotic pion wave function, respecting all theoretical constraints, provides a consistent and theoretically satisfying description of the $\pi\gamma$ and the pion's electromagnetic form factor and is compatible with the pion's valence quark distribution function. The pion's electromagnetic form factor is controlled by soft physics (which can be modelled as the Feynman contribution for the asymptotic wave function) in the experimentally accessible range of momentum transfer in contrast to the $\pi\gamma$ transition form factor which is dominated by the perturbative contribution. We note that similar observations about the smallness of the perturbative contributions and the dominance of the Feynman contributions have been made in the case of the nucleon's form factor [28]. Of course, the present quality of the data does not allow to pin down the form of the wave function exactly. Little modifications of the asymptotic wave function can not be excluded as yet. On the other hand, the CZ wave function is in conflict with the data and ought to be discarded. This is also true for other strongly end-point concentrated wave functions. The use of such wave functions in the analyses of other exclusive reactions involving pions, e.g. $\gamma\gamma \to \pi\pi$ or $B \to \pi\pi$, seems to be unjustified (if one accepts the process-independence of the wave function) and likely leads to overestimates of the perturbative contributions.

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[5] At large Q^2 the Feynman contribution is suppressed by $1/Q^2$ as compared to the perturbative contribution. The latter dominates the elastic form factor only for $Q^2 \gtrsim 50$ GeV^2. This value of Q^2 is, however, very sensitive to the end-point behaviour of the wave function, little modifications may change it considerably.

The next-to-leading order leading twist analysis of the $F_{\pi\gamma}$ form factor (possible for $Q^2 \leq 3\,\mathrm{GeV}^2$) also reveals that the wave function or better the distribution amplitude in that case, is close to the asymptotic one but a little narrower than it. This result is to be contrasted with the modified perturbative approach where the asymptotic wave function works well; the required Q^2-dependent suppression is provided by the Sudakov factor and the transverse degrees of freedom. In any case a systematic next-to-leading order analysis of exclusive reactions involving pions is required.

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Figure captions

Fig. 1. The scaled $\pi\gamma$ transition form factor vs. Q^2. The solid (dashed) line represents the results obtained with the modified perturbative approach using the asymptotic (CZ) wave function. The evolution of the CZ wave function is taken into account. The dotted line represents the limiting behaviour $\sqrt{2}f_\pi$. Data are taken from [8, 9].

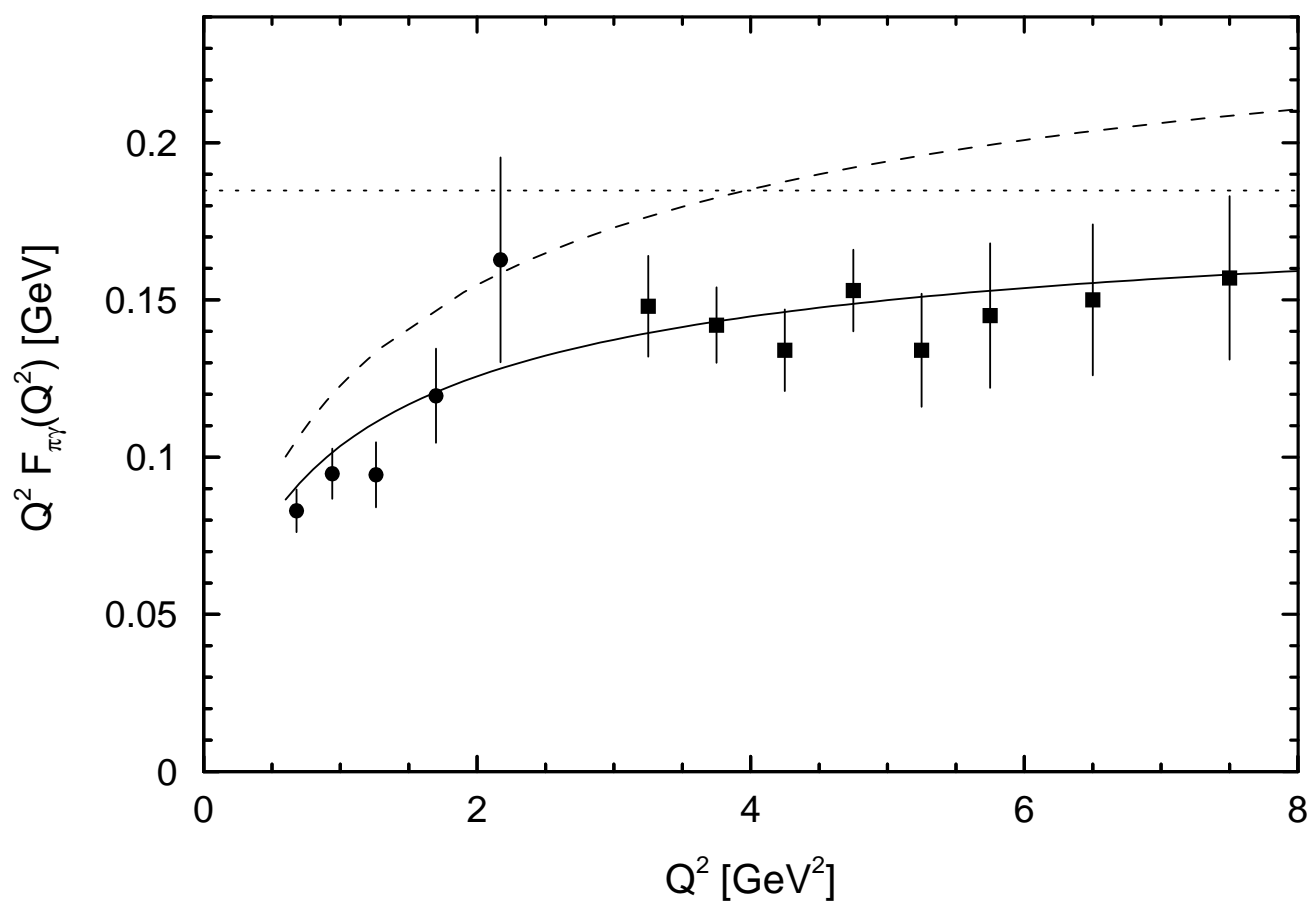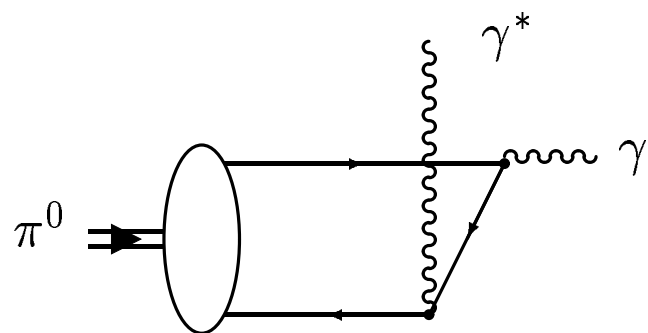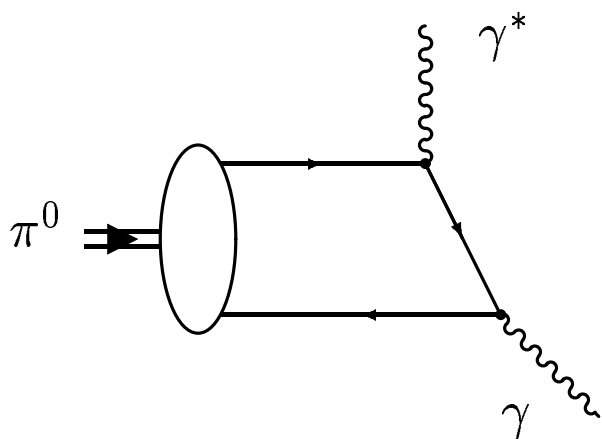Fig. 2. The basic graphs for the $\pi\gamma$ transition form factor.

Fig. 1

Fig. 2